Discrete Mathematics

MATH 211–Fall 2017 Workbook

Written by Breanne Garrett

Edited by Kiera MacPherson, Elizabeth Shaffer, and Ranthony A.C. Edmonds

INTRODUCTION

What is discrete? I'll get to that answer soon, but first I want to tell you about how this course may be different than other math classes you have taken. First, the course will cover math content that strays from the traditional arithmetic, geometry, and algebra that school mathematics is often comprised of. You will see that the topics that fall in the discrete category are much different than you may be used to. Often they will seem like a whole new language and not really math at all. That may seem intimidating, but I hope instead you will find that this different kind of math can be fun and useful.

This course will be using a modified version of the Inquiry-Based Learning (IBL) technique as the mode of delivery. On the next page is an image from the Academy of Inquiry-Based Learning website that gives a loose definition of IBL. In an article written by three of the top IBL math instructors, Inquiry-Based Learning is described as a form of active learning consisting of two pillars: 1) deep engagement in rich mathematics and 2) opportunities to collaborate (Ernst, Hodge, & Yoshinobu, 2017). Students will often work actively on challenging mathematical tasks and through this process will develop skills and ideas that go beyond the rote memory work expected in traditional settings. The result is that students will see the purpose of the mathematics and retain the ideas better for future application. The research also concludes that collaboration on many levels is a necessary part of true mathematical learning and problem solving. Communicating individual ideas as well as listening to and evaluating the methods of others helps to shape our thinking and process it for quicker retrieval. A list of useful websites, articles and other resources backing up the preceding claims is available at the end of this section.

Throughout your workbook you will find tips and strategies, provided by former students, in boxes such as this.

You are likely taking this course because it is a program requirement for your major. That means that the content is necessary for you to understand and apply in your area of expertise. If the idea of trying an alternate approach to mathematics and actively engaging in problem-solving and class discussions doesn't seem appealing, you may want to seriously reconsider your choice of profession. STEM (Science, Technology, Engineering, and Mathematics) is quite the buzz right now, for good reason, and it goes beyond these four strands of academics. To be a part of the growing STEM field, innovative thinking, creativity, self-drive, and effective team skills are vital. One of the overarching goals of this course is to give you experience in these types of skills and, in my opinion, the best mode of delivery of mathematics is IBL. Please keep an open mind this semester, ask any and all questions you have without fear of judgement, and be willing to participate in some productive struggle. I believe you will find this type of learning to be the most rewarding educational experience you've had.

Student Tip: For many of you this will be the first time that you are taking a math class like this one. In some ways that makes it more challenging. However, I assure you as a survivor of the course myself I will tell you it is possible! Take a break if you are really stuck. Coming back to a problem later can often help you find a different approach.

This book is broken into sections that often focus on a real-world task. Throughout the reading you will find, in bold, labels for the exercises that you are to complete. When you come across one of these, please take time to pause your reading and work on the exercise. You may not complete it entirely during the first read through and that's okay – just be sure to take time to think about how you will find a solution. By the end of the section, you will hopefully learn enough to go back and successfully work through the opening task and complete all exercises.

Figure 1. Inquiry-Based Learning for College Mathematics. Courtesy of the Academy of Inquiry-Based Learning. Retrieved from http://www.inquirybasedlearning.org/about/#/what-is-ibl/

So what is discrete mathematics? Discrete math is a collection of branches of mathematics that operate on discrete sets of objects. This means that the values we work with are often countable (but still could be infinite) and can easily be represented with models. Continuous sets of objects, on the other hand, are things like the set of all real numbers. One of the easiest ways to remember the difference between discrete and continuous data is to think about apples and applesauce. Here is a question that will result in a discrete answer every time: "How many apples are on each of the trees in the orchard?" Are you ever going to reply that there are 49.4 apples on a tree? I hope not. Your answers will always be a Natural (counting) number like 49 or 50. To contrast this, a question that will result in a continuous answer is: "How many quarts of applesauce did the apples on this tree produce?" Now, an answer such as 49.4 or even 49.42 quarts is totally appropriate, depending on the tool used to measure the amount of applesauce. Discrete sets of data are distinct and can often be represented by listing the data values, where continuous data cannot be listed since between any two values, an infinite number of other values exist.

An important theme occurring in discrete mathematics is logical thinking. Years ago, before computers really existed, mathematics and related majors often took courses from the philosophy department of a college that focused on logic and reasoning. Now, philosophy courses rarely exist so the reasoning necessary to work in mathematical logic is included in discrete math courses. A deep understanding of logic can make programming or fixing a machine much easier since machines 'think' using mathematical logic.

Now let's do some math!

Exercise 0.1: Four people are waiting in a college admissions office, hoping to finalize their paperwork on student housing, aid, etc. Each has a card on top of their papers that state on one side if they will be living on campus and on the other, their current vaccinations. At this college, it is mandatory that if a student will be living in campus housing, he or she must have received the meningitis vaccine. The admissions secretary can see one side the card of each as they wait. A condensed version of what she sees is below. What is the fewest number of cards she needs to flip over in order to determine whether or not the student is compliant with the housing rule? Which cards? Justify your answer.

DORMITORY	OFF-CAMPUS	MENINGITIS	TETANUS

The preceding exercise asked you to justify your answer. Please get in the habit of doing this, regardless if you are prompted to or not. There may be instances where you will explain your thinking to the class and a variety of methods of solutions will be solicited. Come along with us on this journey to uncover some interesting mathematical topics.

Student Tip: The materials here are made to help guide you through the course work, but if they are not enough I encourage you to work with classmates, look for extra examples as needed and ask for help when you need it. Good luck and have fun! Some of the topics you dive into here may surprise you with how interesting and applicable they are to many real world topics.

The following link includes some interesting videos that promote thinking and learning that is aligned with IBL:
http://theiblblog.blogspot.com/2016/06/10-videos-on-productive-failure-playlist.html

References

Ernst, D. C., Hodge, A., & Yoshinobu, S. (2017). What is Inquiry-Based Learning? *Notices of the American Mathematical Society, 64, (6)*. Retrieved from http://www.ams.org/publications/journals/notices/201706/rnoti-p570.pdf

Fleron, J. & Hotchkiss, P. (2014). What is Inquiry-Based Learning? *Discovering the Art of Mathematics: Mathematical inquiry in the liberal arts.* Retrieved from https://www.artofmathematics.org/blogs/jfleron/what-is-inquiry-based-learning

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Yoshinobu, S. (2017). Inquiry-Based Learning for college mathematics [image]. Retrieved with permission from http://www.inquirybasedlearning.org/about/#/what-is-ibl/