The Product and the Quotient Rule

The Product Rule

We previously learned that,

$$\Big(f + g\Big)' = f' + g'$$

and

$$\Big(f - g\Big)' = f' - g'$$

where f and g are two differentiable functions, i.e. they have a derivative.

The Product Rule

Is it true that $\left(fg\right)' = f'g'$?

The Product Rule

Consider $f(x) = x^4$ and $g(x) = x^3$.

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Consider $f(x) = x^4$ and $g(x) = x^3$. Then,

$$\left(fg\right)' = \left(x^4 \cdot x^3\right)'$$

The Product Rule

Consider $f(x) = x^4$ and $g(x) = x^3$. Then,

$$\begin{aligned} \left(fg\right)' &= \left(x^4 \cdot x^3\right)' \\ &= \left(x^7\right)' \end{aligned}$$

The Product Rule

Consider $f(x) = x^4$ and $g(x) = x^3$. Then,

$$\begin{aligned} \left(fg\right)' &= \left(x^4 \cdot x^3\right)' \\ &= \left(x^7\right)' \\ &= 7x^6 \end{aligned}$$

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$$\begin{aligned} \left(fg\right)' &= \left(x^4 \cdot x^3\right)' \\ &= \left(x^7\right)' \\ &= 7x^6 \end{aligned}$$

But,

$$f'g' = (4x^3)(3x^2)$$

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$$\begin{aligned} \left(fg\right)' &= \left(x^4 \cdot x^3\right)' \\ &= \left(x^7\right)' \\ &= 7x^6 \end{aligned}$$

But,

$$\begin{aligned} f'g' &= (4x^3)(3x^2) \\ &= 12x^5. \end{aligned}$$

The Product Rule

In general $\left(fg\right)' \neq f'g'$.

The Product Rule

In general $\left(fg\right)' \neq f'g'$.

There is a formula that tells us how to find $\left(fg\right)'$ called the product rule.

The Product Rule Formula

If f and g are differentiable functions, the derivative of their product fg is given by the following formula, called the **product rule**:

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If f and g are differentiable functions, the derivative of their product fg is given by the following formula, called the **product rule**:

$$\left(fg\right)' = fg' + gf'$$

The Product Rule Examples

Example 1

$$y = \underbrace{x^2}_{f}\underbrace{e^x}_{g}$$

The Product Rule Examples

Example 1

$$y = \underbrace{x^2}_{\color{blue}{f}}\underbrace{e^x}_{\color{red}{g}}$$

$$f' = 2x$$

The Product Rule Examples

Example 1

$$
\begin{aligned}
y &= \underbrace{x^2}_{f}\underbrace{e^x}_{g} \\
f' &= 2x \\
g' &= e^x
\end{aligned}
$$

The Product Rule Examples

Example 1

$$\begin{aligned} y &= \underbrace{x^2}_{\color{blue}f}\underbrace{e^x}_{\color{red}g} \\ f' &= 2x \\ g' &= e^x \end{aligned}$$

$$y' = \left({\color{blue}f}{\color{red}g}\right)'$$

The Product Rule Examples

Example 1

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\begin{aligned}
y &= \underbrace{x^2}_{\color{blue}f}\underbrace{e^x}_{\color{red}g} \\
f' &= 2x \\
g' &= e^x
\end{aligned}
$$

$$
\begin{aligned}
y' &= \left({\color{blue}f}{\color{red}g}\right)' \\
&= {\color{blue}f}g' + {\color{red}g}f'
\end{aligned}
$$

The Product Rule Examples

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y &= \underbrace{x^2}_{f}\underbrace{e^x}_{g} \\
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$$

$$
\begin{aligned}
y' &= \Big(fg\Big)' \\
&= fg' + gf' \\
&= (x^2)(e^x) + (e^x)(2x)
\end{aligned}
$$

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\begin{aligned}
y' &= \Big(fg\Big)' \\
&= fg' + gf' \\
&= (x^2)(e^x) + (e^x)(2x) \\
&= x^2e^x + 2xe^x
\end{aligned}
$$

The Product Rule Examples

Example 2

$$y = \underbrace{\left(1 - \frac{1}{x}\right)}_{f} \underbrace{\left(x^5 + 1\right)}_{g}$$

The Product Rule Examples

Example 2

$$y = \underbrace{\left(1 - \frac{1}{x}\right)}_{f}\underbrace{\left(x^5 + 1\right)}_{g}$$

$$f' = -(-x)^{-2} = x^{-2} = \frac{1}{x^2}$$

The Product Rule Examples

Example 2

$$y = \underbrace{\left(1 - \frac{1}{x}\right)}_{\color{blue}{f}} \underbrace{\left(x^5 + 1\right)}_{\color{red}{g}}$$

$$f' = -(-x)^{-2} = x^{-2} = \frac{1}{x^2}$$

$$g' = 5x^4$$

The Product Rule Examples

Example 2 (cont'd)

$$\begin{aligned} f' &= -(-x)^{-2} = x^{-2} = \frac{1}{x^2} \\ g' &= 5x^4 \end{aligned}$$

The Product Rule Examples

Example 2 (cont'd)

$$f' = -(-x)^{-2} = x^{-2} = \frac{1}{x^2}$$

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$$y' = \left(fg\right)'$$

The Product Rule Examples

Example 2 (cont'd)

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The Product Rule Examples

Example 2 (cont'd)

$$\begin{aligned} f' &= -(-x)^{-2} = x^{-2} = \frac{1}{x^2} \\ g' &= 5x^4 \end{aligned}$$

$$\begin{aligned} y' &= \left(fg\right)' \\ &= fg' + gf' \\ &= \left(1 - \frac{1}{x}\right)\left(5x^4\right) + \left(x^5 + 1\right)\left(\frac{1}{x^2}\right) \end{aligned}$$

The Product Rule Examples

Example 2 (cont'd)

$$
\begin{aligned}
f' &= -(-x)^{-2} = x^{-2} = \frac{1}{x^2} \\
g' &= 5x^4
\end{aligned}
$$

$$
\begin{aligned}
y' &= \Big(fg\Big)' \\
&= fg' + gf' \\
&= \left(1 - \frac{1}{x}\right)\left(5x^4\right) + \left(x^5 + 1\right)\left(\frac{1}{x^2}\right) \\
&= 5x^4 - 5x^3 + x^3 + \frac{1}{x^2}
\end{aligned}
$$

The Product Rule Examples

Example 2 (cont'd)

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\begin{aligned}
f' &= -(-x)^{-2} = x^{-2} = \frac{1}{x^2} \\
g' &= 5x^4
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$$

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\begin{aligned}
y' &= \left(fg\right)' \\
&= fg' + gf' \\
&= \left(1 - \frac{1}{x}\right)\left(5x^4\right) + \left(x^5 + 1\right)\left(\frac{1}{x^2}\right) \\
&= 5x^4 - 5x^3 + x^3 + \frac{1}{x^2} \\
&= 5x^4 - 4x^3 + \frac{1}{x^2}
\end{aligned}
$$

The Product Rule Examples

Example 3

$$y = \underbrace{-3x}_{f}\underbrace{(x^8 + 6)}_{g}$$

The Product Rule Examples

Example 3

$$y = \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g}$$

$$f' = -3$$

The Product Rule Examples

Example 3

$$\begin{aligned} y &= \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g} \\ f' &= -3 \\ g' &= 8x^7 \end{aligned}$$

The Product Rule Examples

Example 3

$$
\begin{aligned}
y &= \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g} \\
f' &= -3 \\
g' &= 8x^7 \\
y' &= \left(fg\right)'
\end{aligned}
$$

The Product Rule Examples

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$$
\begin{aligned}
y &= \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g} \\
f' &= -3 \\
g' &= 8x^7 \\
\\
y' &= \Big(fg\Big)' \\
&= fg' + gf'
\end{aligned}
$$

The Product Rule Examples

Example 3

$$y = \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g}$$
$$f' = -3$$
$$g' = 8x^7$$

$$y' = \Big(fg\Big)'$$
$$= fg' + gf'$$
$$= (-3x)(8x^7) + (x^8+6)(-3)$$

The Product Rule Examples

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\begin{aligned}
y &= \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g} \\
f' &= -3 \\
g' &= 8x^7 \\
\\
y' &= \left(fg\right)' \\
&= fg' + gf' \\
&= (-3x)(8x^7) + (x^8+6)(-3) \\
&= -24x^8 - 3x^8 - 18
\end{aligned}
$$

The Product Rule Examples

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y &= \underbrace{-3x}_{f}\underbrace{(x^8+6)}_{g} \\
f' &= -3 \\
g' &= 8x^7 \\
\\
y' &= \left(fg\right)' \\
&= fg' + gf' \\
&= (-3x)(8x^7) + (x^8+6)(-3) \\
&= -24x^8 - 3x^8 - 18 \\
&= -27x^8 - 18
\end{aligned}
$$

The Product Rule Examples

Note that in *Example 3*,

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$$y = -3x(x^8 + 6)$$

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$$\begin{aligned} y &= -3x(x^8+6) \\ &= -3x^9 - 18x \end{aligned}$$

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Note that in *Example 3*,

$$\begin{aligned} y &= -3x(x^8+6) \\ &= -3x^9 - 18x \end{aligned}$$

$$y' = -27x^8 - 18$$

The Product Rule Examples

Example 4

$$y = (x^{-2} + x^{-3})(x^5 - 2x^2)$$

The Product Rule Examples

Example 4

$$
\begin{aligned}
y &= (x^{-2} + x^{-3})(x^5 - 2x^2) \\
&= (x^{-2})(x^5) + (x^{-2})(-2x^2) + (x^{-3})(x^5) + (x^{-3})(-2x^2)
\end{aligned}
$$

The Product Rule Examples

Example 4

$$
\begin{aligned}
y &= (x^{-2} + x^{-3})(x^5 - 2x^2) \\
&= (x^{-2})(x^5) + (x^{-2})(-2x^2) + (x^{-3})(x^5) + (x^{-3})(-2x^2) \\
&= x^3 - 2 + x^2 - 2x^{-1}
\end{aligned}
$$

The Product Rule Examples

Example 4

$$\begin{aligned} y &= (x^{-2}+x^{-3})(x^5-2x^2) \\ &= (x^{-2})(x^5)+(x^{-2})(-2x^2)+(x^{-3})(x^5)+(x^{-3})(-2x^2) \\ &= x^3-2+x^2-2x^{-1} \end{aligned}$$

$$y' = 3x^2+2x+2x^{-2}$$

The Quotient Rule

$$\text{Does} \left(\frac{f}{g}\right)' = \frac{f'}{g'}?$$

The Quotient Rule

Consider $f(x) = 7$ and $g(x) = \sqrt{x}$.

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Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

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Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

$$\left(\frac{f}{g}\right)' = \left(\frac{7}{\sqrt{x}}\right)'$$

The Quotient Rule

Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

$$\begin{aligned} \left(\frac{f}{g}\right)' &= \left(\frac{7}{\sqrt{x}}\right)' \\ &= \left(7x^{-1/2}\right)' \end{aligned}$$

The Quotient Rule

Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

$$\begin{aligned} \left(\frac{f}{g}\right)' &= \left(\frac{7}{\sqrt{x}}\right)' \\ &= \left(7x^{-1/2}\right)' \\ &= -\frac{7}{2}x^{-3/2} \end{aligned}$$

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Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

$$\begin{aligned} \left(\frac{f}{g}\right)' &= \left(\frac{7}{\sqrt{x}}\right)' \\ &= \left(7x^{-1/2}\right)' \\ &= -\frac{7}{2}x^{-3/2} \end{aligned}$$

But,

The Quotient Rule

Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

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But,

$$\frac{f'}{g'} = \frac{0}{\frac{1}{2}x^{-1/2}}$$

The Quotient Rule

Consider $f(x) = 7$ and $g(x) = \sqrt{x}$. Then,

$$\begin{aligned}
\left(\frac{f}{g}\right)' &= \left(\frac{7}{\sqrt{x}}\right)' \\
&= \left(7x^{-1/2}\right)' \\
&= -\frac{7}{2}x^{-3/2}
\end{aligned}$$

But,

$$\begin{aligned}
\frac{f'}{g'} &= \frac{0}{\frac{1}{2}x^{-1/2}} \\
&= 0.
\end{aligned}$$

The Quotient Rule

In general $\left(\frac{f}{g}\right)' \neq \frac{f'}{g'}$.

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In general $\left(\frac{f}{g}\right)' \neq \frac{f'}{g'}$.

There is a formula that tells us how to find $\left(\frac{f}{g}\right)'$ called the quotient rule.

The Quotient Rule Formula

If f and g are differentiable functions, the derivative of their quotient $\frac{f}{g}$ is given by the following formula, called the **quotient rule**:

The Quotient Rule Formula

If f and g are differentiable functions, the derivative of their quotient $\frac{f}{g}$ is given by the following formula, called the **quotient rule**:

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

The Quotient Rule Examples

Example 5

$$y = \frac{\overbrace{2x}^{f}}{\underbrace{4 + x^2}_{g}}$$

The Quotient Rule Examples

Example 5

$$y = \frac{\overbrace{2x}^{f}}{\underbrace{4+x^2}_{g}}$$

$$f' = 2$$

The Quotient Rule Examples

Example 5

$$\begin{aligned} y &= \frac{\overbrace{2x}^{f}}{\underbrace{4+x^2}_{g}} \\ f' &= 2 \\ g' &= 2x \end{aligned}$$

The Quotient Rule Examples

Example 5 (cont'd)

$$\begin{aligned} f' &= 2 \\ g' &= 2x \end{aligned}$$

The Quotient Rule Examples

Example 5 (cont'd)

$$\begin{aligned} f' &= 2 \\ g' &= 2x \end{aligned}$$

$$y' = \frac{gf' - fg'}{g^2}$$

The Quotient Rule Examples

Example 5 (cont'd)

$$\begin{aligned} f' &= 2 \\ g' &= 2x \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ y' &= \frac{(4+x^2)(2) - (2x)(2x)}{(4+x^2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 5 (cont'd)

$$\begin{aligned} f' &= 2 \\ g' &= 2x \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ y' &= \frac{(4+x^2)(2) - (2x)(2x)}{(4+x^2)^2} \\ y' &= \frac{8 + 2x^2 - 4x^2}{(4+x^2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 5 (cont'd)

$$
\begin{aligned}
f' &= 2 \\
g' &= 2x
\end{aligned}
$$

$$
\begin{aligned}
y' &= \frac{gf' - fg'}{g^2} \\
y' &= \frac{(4+x^2)(2) - (2x)(2x)}{(4+x^2)^2} \\
y' &= \frac{8 + 2x^2 - 4x^2}{(4+x^2)^2} \\
y' &= \frac{8 - 2x^2}{(4+x^2)^2}
\end{aligned}
$$

The Quotient Rule Examples

Example 6

$$y = \frac{\overbrace{e^x - 1}^{f}}{\underbrace{3x + 2}_{g}}$$

The Quotient Rule Examples

Example 6

$$
\begin{aligned}
y &= \frac{\overbrace{e^x - 1}^{f}}{\underbrace{3x + 2}_{g}} \\
f' &= e^x
\end{aligned}
$$

The Quotient Rule Examples

Example 6

$$
\begin{aligned}
y &= \frac{\overbrace{e^x - 1}^{f}}{\underbrace{3x + 2}_{g}} \\
f' &= e^x \\
g' &= 3
\end{aligned}
$$

The Quotient Rule Examples

Example 6 (cont'd)

$$\begin{aligned} f' &= e^x \\ g' &= 3 \end{aligned}$$

The Quotient Rule Examples

Example 6 (cont'd)

$$\begin{aligned} f' &= e^x \\ g' &= 3 \end{aligned}$$

$$y' = \frac{gf' - fg'}{g^2}$$

The Quotient Rule Examples

Example 6 (cont'd)

$$\begin{aligned} f' &= e^x \\ g' &= 3 \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ y' &= \frac{(3x+2)(e^x) - (e^x - 1)(3)}{(3x+2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 6 (cont'd)

$$\begin{aligned} f' &= e^x \\ g' &= 3 \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ y' &= \frac{(3x+2)(e^x) - (e^x - 1)(3)}{(3x+2)^2} \\ y' &= \frac{3xe^x + 2e^x - 3e^x + 3}{(3x+2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 6 (cont'd)

$$\begin{aligned} f' &= e^x \\ g' &= 3 \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ y' &= \frac{(3x+2)(e^x) - (e^x - 1)(3)}{(3x+2)^2} \\ y' &= \frac{3xe^x + 2e^x - 3e^x + 3}{(3x+2)^2} \\ y' &= \frac{3xe^x - e^x + 3}{(3x+2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 7

$$y = \frac{\overbrace{x^3 - 2x}^{f}}{\underbrace{x^2}_{g}}$$

The Quotient Rule Examples

Example 7

$$y = \frac{\overbrace{x^3 - 2x}^{f}}{\underbrace{x^2}_{g}}$$

$$f' = 3x^2 - 2$$

The Quotient Rule Examples

Example 7

$$y = \frac{\overbrace{x^3 - 2x}^{f}}{\underbrace{x^2}_{g}}$$

$$f' = 3x^2 - 2$$

$$g' = 2x$$

The Quotient Rule Examples

Example 7 (cont'd)

$$\begin{aligned} f' &= 3x^2 - 2 \\ g' &= 2x \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$\begin{aligned} f' &= 3x^2 - 2 \\ g' &= 2x \end{aligned}$$

$$y' = \frac{gf' - fg'}{g^2}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$\begin{aligned} f' &= 3x^2 - 2 \\ g' &= 2x \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ &= \frac{(x^2)(3x^2 - 2) - (x^3 - 2x)(2x)}{(x^2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$\begin{aligned} f' &= 3x^2 - 2 \\ g' &= 2x \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ &= \frac{(x^2)(3x^2 - 2) - (x^3 - 2x)(2x)}{(x^2)^2} \\ &= \frac{3x^4 - 2x^2 - 2x^4 + 4x^2}{(x^2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$\begin{aligned} f' &= 3x^2 - 2 \\ g' &= 2x \end{aligned}$$

$$\begin{aligned} y' &= \frac{gf' - fg'}{g^2} \\ &= \frac{(x^2)(3x^2 - 2) - (x^3 - 2x)(2x)}{(x^2)^2} \\ &= \frac{3x^4 - 2x^2 - 2x^4 + 4x^2}{(x^2)^2} \\ &= \frac{x^4 - 2x^2}{(x^2)^2} \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

Note that,

The Quotient Rule Examples

Example 7 (cont'd)

Note that,

$$y = \frac{x^3 - 2x}{x^2}$$

The Quotient Rule Examples

Example 7 (cont'd)

Note that,

$$\begin{aligned} y &= \frac{x^3 - 2x}{x^2} \\ &= \frac{x^3}{x^2} - \frac{2x}{x^2} \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

Note that,

$$
\begin{aligned}
y &= \frac{x^3 - 2x}{x^2} \\
&= \frac{x^3}{x^2} - \frac{2x}{x^2} \\
&= x - \frac{2}{x}.
\end{aligned}
$$

The Quotient Rule Examples

Example 7 (cont'd)

Note that,

$$
\begin{aligned}
y &= \frac{x^3 - 2x}{x^2} \\
&= \frac{x^3}{x^2} - \frac{2x}{x^2} \\
&= x - \frac{2}{x}.
\end{aligned}
$$

Then

$$
y' = 1 + \frac{2}{x^2}
$$

The Quotient Rule Examples

Example 7 (cont'd)

$$y' = 1 + \frac{2}{x^2}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$y' = 1 + \frac{2}{x^2}$$

$$y' = \frac{x^4 - 2x^2}{(x^2)^2}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$y' = 1 + \frac{2}{x^2}$$

$$\begin{aligned} y' &= \frac{x^4 - 2x^2}{(x^2)^2} \\ &= \frac{x^4 - 2x^2}{x^4} \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$y' = 1 + \frac{2}{x^2}$$

$$\begin{aligned} y' &= \frac{x^4 - 2x^2}{(x^2)^2} \\ &= \frac{x^4 - 2x^2}{x^4} \\ &= \frac{x^4}{x^4} - \frac{2x^2}{x^4} \end{aligned}$$

The Quotient Rule Examples

Example 7 (cont'd)

$$y' = 1 + \frac{2}{x^2}$$

$$\begin{aligned} y' &= \frac{x^4 - 2x^2}{(x^2)^2} \\ &= \frac{x^4 - 2x^2}{x^4} \\ &= \frac{x^4}{x^4} - \frac{2x^2}{x^4} \\ &= 1 + \frac{2}{x^2} \end{aligned}$$

The Quotient Rule Examples

Example 8

$$y = \frac{3x^2 + 2\sqrt{x}}{x}$$

The Quotient Rule Examples

Example 8

$$\begin{aligned} y &= \frac{3x^2 + 2\sqrt{x}}{x} \\ &= \frac{3x^2}{x} + \frac{2x^{1/2}}{x} \end{aligned}$$

The Quotient Rule Examples

Example 8

$$\begin{aligned} y &= \frac{3x^2 + 2\sqrt{x}}{x} \\ &= \frac{3x^2}{x} + \frac{2x^{1/2}}{x} \\ &= 3x + 2x^{-1/2} \end{aligned}$$

The Quotient Rule Examples

Example 8

$$\begin{aligned} y &= \frac{3x^2 + 2\sqrt{x}}{x} \\ &= \frac{3x^2}{x} + \frac{2x^{1/2}}{x} \\ &= 3x + 2x^{-1/2} \end{aligned}$$

$$y' = 3 - x^{-3/2}$$

The Quotient Rule Examples

Example 9

$$y = \frac{7e^x}{x^6}$$

The Quotient Rule Examples

Example 9

$$\begin{aligned} y &= \frac{7e^x}{x^6} \\ &= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g} \end{aligned}$$

The Quotient Rule Examples

Example 9

$$y = \frac{7e^x}{x^6}$$

$$= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g}$$

$$f' = 7e^x$$

The Quotient Rule Examples

Example 9

$$
\begin{aligned}
y &= \frac{7e^x}{x^6} \\
&= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g} \\
f' &= 7e^x \\
g' &= -6x^{-7}
\end{aligned}
$$

The Quotient Rule Examples

Example 9

$$
\begin{aligned}
y &= \frac{7e^x}{x^6} \\
&= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g} \\
f' &= 7e^x \\
g' &= -6x^{-7} \\
\\
y' &= fg' + gf'
\end{aligned}
$$

The Quotient Rule Examples

Example 9

$$\begin{aligned} y &= \frac{7e^x}{x^6} \\ &= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g} \\ f' &= 7e^x \\ g' &= -6x^{-7} \end{aligned}$$

$$\begin{aligned} y' &= fg' + gf' \\ &= (7e^x)(-6x^{-7}) + (x^{-6})(7e^x) \end{aligned}$$

The Quotient Rule Examples

Example 9

$$
\begin{aligned}
y &= \frac{7e^x}{x^6} \\
&= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g} \\
f' &= 7e^x \\
g' &= -6x^{-7}
\end{aligned}
$$

$$
\begin{aligned}
y' &= fg' + gf' \\
&= (7e^x)(-6x^{-7}) + (x^{-6})(7e^x) \\
&= -42x^{-7}e^x + 7x^{-6}e^x
\end{aligned}
$$

The Quotient Rule Examples

Example 9

$$\begin{aligned}
y &= \frac{7e^x}{x^6} \\
&= \underbrace{(7e^x)}_{f}\underbrace{(x^{-6})}_{g} \\
f' &= 7e^x \\
g' &= -6x^{-7} \\
\\
y' &= fg' + gf' \\
&= (7e^x)(-6x^{-7}) + (x^{-6})(7e^x) \\
&= -42x^{-7}e^x + 7x^{-6}e^x \\
&= -\frac{42e^x}{x^7} + \frac{7e^x}{x^6}
\end{aligned}$$