

My research area is in a branch of abstract algebra that focuses on mathematical objects called rings. A classic example of a ring is the set of integers. If we take any two integers, for example 2 and 3, we know that  $2 \cdot 3 = 3 \cdot 2$ , which shows us that multiplication is commutative. Thus, the integers are a commutative ring. Also, if we take the product of any two integers, call them  $a$  and  $b$ , if their product  $a \cdot b = 0$  we know that  $a$  or  $b$  must be 0. Any ring that possesses this property is called an integral domain. If there exist two nonzero elements, however, whose product is zero we call such elements zero divisors. In particular, I study factorization in commutative rings with zero divisors.

Factorization theory is concerned with the decomposition of mathematical objects and its applications are far reaching. Such an object could be a polynomial, a number in the set of integers, or more generally an element in a ring. When we factor an element we reduce it to a product of its basic building blocks called irreducible elements. For example, at an elementary level, cryptography is study of factoring very large numbers into the products of building blocks called prime numbers. For factorization in integral domains authors agree on definitions for the simplest elements of these rings, such as primes and irreducibles, as well as certain ring theoretic properties like unique factorization that stem from these definitions. Thus many authors have shifted to generalizing this theory to commutative rings with zero divisors. This theory is much less uniform. The presence of zero divisors has led different authors to establish different definitions of irreducible elements, which in turn leads to different factorization techniques based on what the basic building blocks are considered to be.

The purpose of this thesis is to create an overarching theory for factorization in polynomial rings with zero divisors. I aim to do this by characterizing when a polynomial ring satisfies certain ring theoretic properties, such as unique factorization, half-factorization, bounded factorization, finite factorization, the ascending chain condition on principal ideals, and atomicity. In particular, I consider the following question for a commutative ring with identity  $R$  and its polynomial extension ring  $R[X]$  : if one of these rings has a certain factorization property, does the other? If the answer is no, what conditions must be in place for the answer to be yes? If there are no suitable conditions, are there counterexamples that demonstrate a polynomial can possess one factorization property and not another? My dissertation will serve as a primary source for a thorough treatment of factorization in polynomial rings with zero divisors.